TNCore

Mathematics Task Arcs

Overview of Mathematics Task Arcs:

A task arc is a set of related lessons which consists of eight tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Common Core State Standards for Mathematics. In some cases, a small number of related standards from more than one domain may be addressed.

A unique aspect of the task arc is the identification of essential understandings of mathematics. An essential understanding is the underlying mathematical truth in the lesson. The essential understandings are critical later in the lesson guides, because of the solution paths and the discussion questions outlined in the share, discuss, and analyze phase of the lesson are driven by the essential understandings.

The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however the context or representations change.

Bias and sensitivity:

These math task arcs are peer-reviewed and have been vetted for content by experts. However, it is the responsibility of local school districts to review these units for social, ethnic, racial, and gender bias before use in local schools.

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Grade 7

Investigating Inequalities

A SET OF RELATED LESSONS

UNIVERSITY OF PITTSBURGH

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Investigating Inequalities

A SET OF RELATED LESSONS

Overview

In this set of related lessons, students extend their previous understanding of equations to include inequalities.

In Task 1, students are provided a context in which to familiarize themselves with the idea of having more than one answer. In Task 2, students solve an inequality problem situation in a variety of ways. In Task 3, students are explicitly asked to set up and solve an inequality with a positive rate of change algebraically. In Task 4, students solidify their understanding of what it means to be a solution to an inequality, the differences between equations and inequalities, and algebraic methods for solving inequalities.

In Task 5, students encounter a problem situation with a negative rate of change. They analyze several methods for solving the problem, but note that their previous algebraic method does not work, prompting further study in Tasks 6 and 7 of why the inequality must be "switched' when multiplying or dividing by a negative number. In Task 8, students solidify their understanding of inequalities with negative rates of change.

The tasks are aligned to the 7.EE.B.4, 7.EE.B.4a, and 7.EE.B.4b Content Standards of the CCSSM.

The prerequisite knowledge necessary to enter these lessons is an understanding of how to solve equations.

Through engaging in the tasks in this set of related lessons, students will:

- compare problem situations that can be solved with equations to problem situations that can be solved with inequalities;
- compare the solution of an equation to the solution set of an inequality;
- solve equations and inequalities using various methods; and
- articulate why it is necessary to "switch" the inequality symbol when dividing or multiplying by a negative value.

By the end of these lessons, students will be able to answer the following overarching questions:

- How are inequalities the same as equations? How are they different?
- How are inequalities used to solve problems that have many different solutions?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning's expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.

Identified CCSSM and Essential Understandings[1]

CCSS for Mathematical Content: Expressions and Equations		**Essential Understandings**
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.		
7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	
7.EE.B.4a	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	Because an equation is a statement that two expressions have the same value, certain properties apply. a. The addition property of equality indicates that adding the same number to both expressions in the equation maintains the balance in the equation. b. The subtraction property of equality indicates that subtracting the same number from both expressions in the equation maintains the balance in the equation. c. The multiplication property of equality indicates that multiplying each expression in the equation by the same non-zero number maintains the balance in the equation. d. The division property of equality indicates that dividing the same non-zero number into both expressions in the equation maintains the balance in the equation.

1 Council of Chief State School Officers (CCSSO) & National Governors Association Center for Best Practices (NGA Center). (2010). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

CCSS for Mathematical Content: Expressions and Equations		Essential Understandings
7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*	An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true. The solution set of an inequality in one variable contains infinitely many values because the real numbers are both infinite and dense. In any given real-world context, not all of the values will make sense. Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context. Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

The CCSS for Mathematical Practice[2]

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

2 Council of Chief State School Officers (CCSSO) & National Governors Association Center for Best Practices (NGA Center). (2010). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

Tasks' CCSSM Alignment

Task	7.EE.B.4	7.EE.B.4a	7.EE.B.4b	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 Fresh Foods Developing Understanding	✓	✓	✓	✓	✓		✓		✓	✓	✓
Task 2 Lifeguard Developing Understanding	✓		✓	✓	✓		✓	✓	✓	✓	
Task 3 Video Game Rental Developing Understanding	✓		✓	✓	✓	✓	✓		✓	✓	
Task 4 The Possibilities are Endless! Solidifying Understanding	✓			✓			✓		✓	✓	✓
Task 5 Deep Dark Secret Developing Understanding	✓		✓	✓	✓		✓	✓	✓	✓	
Task 6 Josie is so Negative These Days! Developing Understanding	✓		✓	✓	✓	✓	✓		✓	✓	✓
Task 7 Flip It Over–This Inequality is Done! Developing Understanding	✓		✓	✓	✓				✓	✓	
Task 8 Solution(s) to this Problem Solidifying Understanding	✓			✓		✓	✓		✓	✓	✓

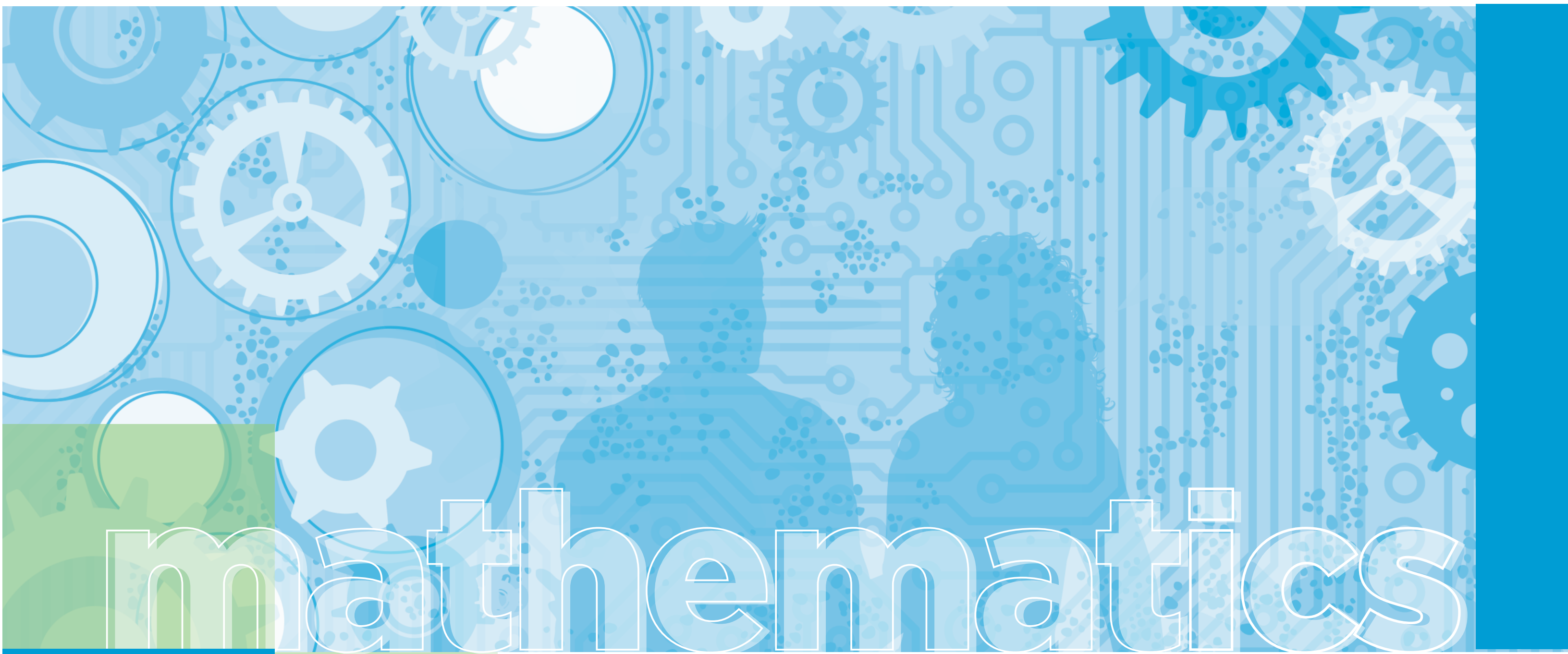
Lesson Progression Chart

Overarching Questions

- How are inequalities the same as equations? How are they different?
- How are inequalities used to solve problems that have many different solutions?

	Task 1 **Fresh Foods** *Developing Understanding*	Task 2 **Lifeguard** *Developing Understanding*	Task 3 **Video Game Rental** *Developing Understanding*	Task 4 **The Possibilities are Endless!** *Solidifying Understanding*
Content	Develop an understanding of inequalities and how they compare to equations. The fact that inequalities have infinitely many solutions will be explored.	Write and solve several inequalities and compare the meaning of each inequality's solution set. Introduction of "at least" contextualizes the unlimited solution set.	Compare and contrast two inequalities that are derived from two different forms of data. Use of the term "less than" has students deepening the understanding of the meaning of the solution set.	Solidify strategies and solution paths for solving inequalities.
Strategy	Students use and compare numerical strategies and an algebraic strategy for solving an inequality. Substitution is used to check the solution set.	Students may use scaling, extending the line, or algebraic methods to solve the equation/ inequality.	Students may create a table of values or use a numeric strategy when solving an inequality, but they are challenged to use algebraic methods, as well.	Students may use substitution but are pressed to solidify an algebraic method to solve the equation and inequalities.
Representations	Starts with a context and a table of values; represent solutions using words and inequality notation.	Starts with a graph; students may use the graph, a table of values, or equations/ inequalities.	Starts with a context and students use an equation/ inequality and possibly a table of values to support the equation/ inequality.	Starts with equations/ inequalities that students solve algebraically.

	Task 5 **Deep Dark Secret** *Developing Understanding*	Task 6 **Josie is so Negative These Days!** *Developing Understanding*	Task 7 **Flip It Over–This Inequality is Done!** *Developing Understanding*	Task 8 **Solution(s) to this Problem** *Solidifying Understanding*
Content	Given a negative rate of change, the previously used algebraic methods lead to an incorrect answer, so students must explore other methods.	Analyze distance and direction of two numbers on the number line to make sense of WHY multiplying or dividing by a negative number "reverses" the inequality.	Analyze two different algebraic solution strategies for inequalities with negative coefficients; determine why it makes sense mathematically to "switch" the inequality symbol in one strategy and not in the other.	Solidify understanding of solving inequalities with negative coefficients.
Strategy	Students use numeric strategies and/or algebraic methods.	Students use substitution, numeric strategies, or direction on the number line.	Students use algebraic strategies and substitution.	Students use algebraic strategies and substitution.
Representations	Begin with a context and use a table of values, graph, and/or an equation or inequality.	Begin with a context and use words, inequalities, and a number line.	Use equations and number lines.	Use equations and number lines, as well as providing a context.



Grade 7

Tasks and Lesson Guides

Investigating Inequalities

A SET OF RELATED LESSONS

TASK 1

Name______________________________

Fresh Foods

Benjamin is considering taking a job at the Fresh Foods grocery store. The store pays its employees a $60 one-time bonus in addition to an hourly pay rate of $8.40. Benjamin is working to save money for the class trip. Airfare, hotel, and food will cost $1800, so he knows that he needs to make more than this amount in order to pay for the trip and have additional spending money during the trip.

Summer Job	Summer Hours
Cashier	250
Stock Person	210
Deli	195
Bakery	180
Bagger	205

1. Which summer job(s) at the Fresh Foods grocery will allow Benjamin to meet his $1800 goal? Show all work and explain your reasoning.

2. Describe the number of hours Benjamin must work at any Fresh Foods position to make:

   a. exactly $1800

   b. more than $1800

   c. less than $1800

   Represent your solutions using words, inequality notation, and a number line.

LESSON GUIDE 1

Fresh Foods

Rationale for Lesson: In a context, students begin to develop an understanding of inequalities and how they compare to equations. They recognize that inequalities have infinitely many solutions, and they represent these solutions using words, inequality notation, and the number line.

Task 1: Fresh Foods

Benjamin is considering taking a job at the Fresh Foods grocery store. The store pays its employees a $60 one-time bonus in addition to an hourly pay rate of $8.40. Benjamin is working to save money for the class trip. Airfare, hotel, and food will cost $1800, so he knows that he needs to make more than this amount in order to pay for the trip and have additional spending money during the trip.

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   a. exactly $1800
   b. more than $1800
   c. less than $1800
   Represent your solutions using words, inequality notation, and a number line.

Common Core Content Standards		
	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4a	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.
Essential Understandings	• An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true. • The solution set of an inequality in one variable contains infinitely many values because the real numbers are both infinite and dense. • Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.
Materials Needed	• Student reproducible task sheet • Calculator

LESSON GUIDE 1

SET-UP PHASE

Please read the task silently before I have somebody read the task aloud. Why is Benjamin saving money, and how much does he need to save? What does the information in the table represent? What is meant by a "one-time bonus"? Why might a company offer signing bonuses? Work independently for about five to ten minutes before bringing your solution strategies to your group.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Group can't get started.	What do you need to figure out in this problem?	How can you determine Benjamin's pay? For example, if Benjamin accepts the job of cashier, how many hours will he work? How much money will he make? Explain how you know.
Question 2: Uses guess-and-check.	Can you explain your method to me? How are you determining the number of hours he must work to make $1800?	In our previous work, we solved equations. How can you represent and solve this problem situation with an equation?
Creates a table of values (or extends the existing table of values).	Can you explain your method to me? How did you choose the values for the table?	Can you solve this problem using an equation? Explain why an equation can or cannot be used.
Sets up an equation and solves it.	How does this equation represent the problem? Can you tell me what each term in your equation represents?	How can you use an equation to solve for the values that are more than or less than $1800?

LESSON GUIDE 1

SHARE, DISCUSS, AND ANALYZE PHASE

EU: An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true.

- Tell us about how you determined which summer jobs Benjamin should take. *(We multiplied the hourly rate, which is $8.40, by the number of hours he can work on a job and then added the $60 bonus.)*
- Who can add on to what this group just shared? *(If the amount is more than $1800, then it is a solution.)*
- Did anybody use a different method? *(We subtracted the one-time bonus and then divided by $8.40 to find that he needs to work 207.14 hours.)*
- What does 207.14 mean in the context of the problem? *(We have to work more than 207.14 hours to make more than $1800.)*
- What equation can we write to figure out the exact number of hours needed to earn $1800? *($60 + $8.40x = $1800)*
- Who can say what the variable x represents? *(The number of hours he works on a job.)*
- We found that x is 207.14. **(Marking)**
- How many of the summer jobs at Fresh Foods meet Benjamin's criteria? *(The cashier and stock person jobs.)*
- I heard one group say that Benjamin can take two jobs. Why are these the only jobs? *(Benjamin can work more than 207.14 hours.)*
- Say more and refer specifically to the amounts in the table. *(He can work 210 and 215 for the cashier and stock person jobs.)*
- Why can't he work the other jobs? (**Challenging**) *(Because you can't work as many as 207.14 hours on those jobs.)*
- How can we check to see if Benjamin can work these jobs? *(We substituted these amounts. He can earn more than $1800 if he does these jobs.)*
- Everyone stop and jot. Show this work.
- How do the two ways of solving this problem differ from each other? *(He subtracted the $60 bonus and then divided. She multiplied.)*
- The previous group found this by multiplying the number of hours and the pay per hour and then adding the signing bonus, while another group determined the number of hours by working backwards–subtracting and then dividing. **(Marking and Revoicing)**
- So I'm hearing you say that two of the jobs described in the table allow Benjamin to earn more than the amount of money needed.
- How would this be written algebraically? *($8.40x + $60 = $1800)*
- Who agrees? Who disagrees?
- Are these the only jobs that Benjamin can work in order to earn $1800? **(Challenging)** *(He can work any job that is more than 207.14 hours.)*
- Why do we have to write $8.40x + $60 > $1800? *(It says he can work any job that ends up more than $1800.)*
- An inequality is a statement comparing the relative magnitude of two expressions, the 8.40*x* + 60 compared with 1800. As a result, we determine if this is true or false. **(Marking and Recapping)**
- What makes up the solution set of this inequality? *(All the hours that Benjamin can work.)*
- All of the values of the variable that make the statement true. **(Revoicing)**

LESSON GUIDE 1

EU: Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context.

- Earlier, there was a disagreement about whether there are just two solutions or there were more than two. How many different jobs might there be? **(Challenging)** *(Infinitely many. Anything with hours greater than 207.14, right?)*
- Who agrees or disagrees? Explain. *(I disagree because it's not like he's going to work a trillion hours.)*
- So let's assume I choose a job with a certain number of hours, say 209–how do you know whether this is a solution? *(You plug it in to see if it gives an answer that is greater than $1800.)*
- You put in the number of hours for the variable. "You plug in the values." So how many solutions might there be? **(Marking and Revoicing)**
- Who can show what was just said on the number line? *(All the values from 208 to the right are solutions.)*
- I heard many different answers and there may be a couple different correct answers here, so I'd like one group to explain why they think there are infinitely many solutions and another group to explain why they disagree. *(All values to the right of 207.14 or about 208 are solutions–this is infinite. But not all solutions make sense here so we know there are a lot but because you can't work infinitely many hours, an infinite number of hours does not make sense.)*
- What I think I hear both groups saying is that inequalities have infinitely many solutions, but you have to take the problem situation into consideration and ask if this is logical, too. **(Marking and Recapping)**
- How is the solution set to an inequality the same/different than the solution(s) to an equation? *(When solving equations we were just looking for one solution. Inequalities may have infinitely many solutions that can make it true.)*
- What in the situation tells you that this is an inequality? *(Benjamin wants to earn more than $1800.)*
- What would it have said if it was an equality situation? *(Benjamin wants to earn $1800.)*
- Great point. Solving inequalities and determining how solving them is the same/different than solving equations is exactly what we're going to study in the next several lessons.

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.
EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Who can say more about this group's strategy? They subtracted 60 and then divided by 8.40? *(They were solving the equation.)*
- When we use this strategy, we are using the properties of equality. Why can we "follow these steps"? What does it help us figure out? Why subtract? Why divide? *(We subtract the bonus because that is an extra amount. We divide because we have to figure out the number of hours, the 207.14.)*
- What does the solution $x = 207.14$ represent? *(The number of hours worked to make exactly $1800.)*
- When we put 207.14 in for the variable in the equation we would write $8.40(207.14) + $60 = $1800. What do we know about each side of the equation? *(Each side is $1800.)*
- Each side of the equation is the same. Does each step maintain the balance? For example, after you subtract $60 from both sides, are these expressions still equal? **(Challenging)** *(Yes, because doing the same thing to both sides keeps them equal.)*
- Who can add on to this? Why solve the equation when we're looking for the solution(s) to an inequality? *(The solution to the equation tells us this is the answer. It tells us the number of hours Benjamin MUST work to earn $1800.)*
- Once we know the solution to the equality, then how can this help you know the other jobs or the solution set? *(Then you can write which number of hours will give him more than $1800.)*
- How does this relate to parts b and c? Do we need to solve new equations or inequalities to answer b and c? Explain. *(No, because we know all of the numbers greater than 207.14 are enough hours and all of the ones lower than 207.14 will not give him enough money.)*
- I'm hearing that the properties of equality can be used to solve the equation. If I show this solution on the number line, the line is split. One portion shows all the values greater than the solution, the 207.14, while the other contains all the values less than the solution, less than 207.14. One of those half-lines is the solution to the inequality. **(Marking and Recapping)**
- Who sees this and understands what I am saying and can come up and point to the number line and say back what I said?
- Who can explain how to determine this balance point, or as somebody in another class called it, the "breaking" point? *(This is the solution to the equation.)*
- So I'm hearing that the solution to the equation 8.40x$ + $60 = $1800 creates this balance point or "breaking" point on the number line. In the case of inequalities, then, solutions to "greater than" will be on one side of this point while solutions to "less than" will be on the other. **(Marking and Revoicing)**

LESSON GUIDE 1

Application	If a competing grocery store pays $9.50 per hour with no bonus, how many hours must Benjamin work to make more than $1800? Show all work and explain your reasoning.
Summary	What is the difference between an equation and an inequality?
Quick Write	Use words and the number line to describe the difference between $x = 5$, $x < 5$, and $x > 5$.

Support for students who are English learners (EL):

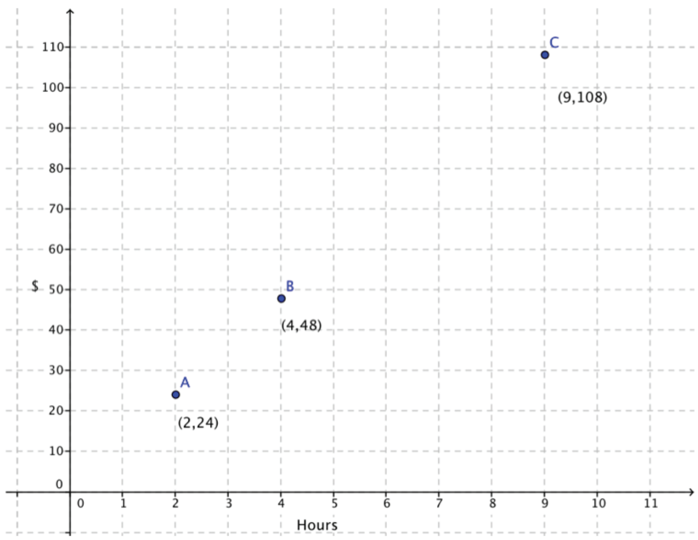
1. Take time during the Set-Up phase and throughout the lesson to assess understanding of vocabulary terms and to discuss the meaning of terms such as *hourly rate*, *one-time bonus*, etc.
2. Private think time during the Explore phase is provided so students have time to organize their thoughts and struggle with the material individually. Cooperative learning is beneficial for all students, but in particular it gives students who are English learners the opportunity to work through ideas in a small group before sharing out to the class.

Name____

TASK 2

Lifeguard

Benjamin's friend, Maleeka, is taking a job as a lifeguard at the city pool. She will make a constant rate per hour. The graph below represents Maleeka's potential earnings for three different numbers of hours of summer work.



1. Maleeka's short-term goal is to earn more than $80 for new games. Describe the number of hours that she must work in order to make enough money for the games. Show all work and represent your answer using words, inequality notation, and a number line.

2. Maleeka's long-term summer goal is to make more than $750. Determine the number of hours that she must work in order to meet this goal. Show all work and represent your answer using words, inequality notation, and a number line.

LESSON GUIDE 2

Lifeguard

Rationale for Lesson: Students will interpret a graph to determine the solution set to an inequality. While the graph is one of several efficient methods that may be used to answer the first question, the second part presses students to search for an efficient method to solve inequalities.

Task 2: Lifeguard
Benjamin's friend, Maleeka, is taking a job as a lifeguard at the city pool. She will make a constant rate per hour. The graph below represents Maleeka's potential earnings for three different numbers of hours of summer work.

1. Maleeka's short-term goal is to earn more than $80 for new games. Describe the number of hours that she must work in order to make enough money for the games. Show all work and represent your answer using words, inequality notation, and a number line.
2. Maleeka's long-term summer goal is to make more than $750. Determine the number of hours that she must work in order to meet this goal. Show all work and represent your answer using words, inequality notation, and a number line.

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure.	

Essential Understandings	• An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality. • Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context.
Materials Needed	• Student reproducible task sheet • Graph paper, straight edge, calculator

LESSON GUIDE 2

SET-UP PHASE

Take a few minutes to read the task silently to yourself before we discuss the problem. Who can say, in their own words, what this task is about without giving away a solution strategy? Has anybody ever done jobs around the house to earn money? Maybe in order to earn allowance money? Did you have a financial goal? What might be some examples of short-term goals? Long-term goals?

EXPLORE PHASE

<table>
<tr><th>Possible Student Pathways</th><th>Assessing Questions</th><th>Advancing Questions</th></tr>
<tr><td>Group can't get started.</td><td>What do the points on the graph represent? How much money does Maleeka want to save for her short-term goal?</td><td>Between what two values will you find the number of hours that Maleeka must work? How can you determine the exact value?</td></tr>
<tr><td>Extends the line and names the point on the line.</td><td>Can you tell me what you found out by extending the line? Where will the solution(s) appear?</td><td>What patterns do you notice on the graph? Your method will work, but can you use the patterns in the numbers to come up with a more efficient method?</td></tr>
<tr><td>Scales up using a table of values.
<table>
<tr><th>Hours</th><th>Dollars</th></tr>
<tr><td>2</td><td>24</td></tr>
<tr><td>4</td><td>48</td></tr>
<tr><td>9</td><td>108</td></tr>
<tr><td>18</td><td>216</td></tr>
<tr><td>36</td><td>432</td></tr>
</table></td><td>Can you explain your method to me? What does your table tell you about the problem?</td><td>Indicate in the table the values that give you the response to the question in the task and explain why you circled the values.</td></tr>
<tr><td>Sets up and solves an inequality.
$12x > 750$</td><td>What does each of the values in the equality represent with respect to the problem situation?</td><td>How many solutions are there to this inequality? Is the solution set for the inequality the same/different than the solution set to the problem situation? Explain.</td></tr>
</table>

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE 2

EU: An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true.

Extending the line:

- Who can explain the values on the graph to us? What important characteristics can you tell us about these values? *(The values are linear. They go up by 12.)*
- Who can restate and then add on to what she just said about the graph?
- How does the graph represent the problem situation? *(Maleeka makes the same amount every hour, which is $12. If you connect the points, they form a line.)*
- Will a member from a group that used the graph to determine the solutions tell us the solution set and how you know the solution set meets Maleeka's short-term goal, the $80? *(She wanted to make $80, so I connected the points so that I could read the $80 point. Anything more than this value meets her short-term goal.)*
- Who can say back how this group used the graph to determine the solution set? *(The $80 wasn't on the graph so I had to connect the dots. Then I used the graph to figure out the x-value when y = $80.)*
- Why are you determining the x value? *(Because on the graph, the x value is the number of hours that Maleeka must work to earn her $80.)*
- I'm hearing that you looked at the relationship between the amount of money on the y-axis and the number of hours on the x-axis (point to the x and y-axis). **(Revoicing)**

A numeric solution path:

- Group B did this a little differently. *(We really did $80/12.)*
- Who knows why he divided by 12? *(She earns $12 per hour, so to work backwards and figure out the hours needed, you have to divide.)*
- You gave us a numeric solution path. **(Revoicing)**

Algebraic solution path:

- Group C used an algebraic approach. Let's hear their thinking. *(We set up and solved an equation, 12x = 80, so anything greater than $80 will meet her goal.)*
- How does $12x = 80$ model the problem that we are trying to solve? *(The 12 represents the number of dollars per hour and the 80 represents the number of dollars that she wants to earn, and the x represents the number of hours.)*
- I don't get it, why are we multiplying 12 by x? **(Challenging)** *(When we multiply the number of hours worked by the amount of dollars she makes per hour, this is how we get the amount of money earned.)*
- You said that 12 times the number of hours gives you the amount earned, $80 is the amount earned, so I can set them equal to each other. **(Marking)**

Solving the problem:

- Group C solved $12x = 80$ to determine the number of hours that Maleeka must work. Remind us again how you solved this equation. *(We divided both sides by 12.)*
- Wait a minute; the numeric solution path also divided $80 by $12 per hour.
- The only difference is that one person wrote an equation, $12x = 80$, and the other one used division, 80/12. Both are valid ways of solving the problem but one is algebraic and one is numeric. **(Marking)**
- What does the solution $6\frac{2}{3}$ tell us about the problem situation? *(It tells us the number of hours that she must work to meet her $80 goal.)*

LESSON GUIDE 2

- As an inequality, we can write $12x > 80$. What does this tell me that is different than $12x = 80$? **(Challenging)** *(We actually want to know when Maleeka will make more than $80.)*
- We are comparing $12x$ to $80 and it is the $12x$ that has to be bigger. The problem says that the $12x$ has to be greater than $80. **(Revoicing and Marking)**
- Remember when we solved equations, there was only one solution to the problem. In this problem, there are many solutions. Can someone come to the board and point to a solution to the inequality? *(8 hours.)*
- How can we use the inequality to figure out if the value that we chose is true for the inequality? *(We can plug it in.)*
- Who can show us what is meant by plugging in? *(12 * 8 = 96, which is more than $80.)*
- And what happens if we substitute 9 in the inequality? Stop and jot, everyone do this. *($108)* What happens with 10 hours? 11 hours?
- Stop and jot. Finish this statement in your notebook: "A value will always be a solution to the inequality $12x > 80$ if____________" *(The value is greater than $6\frac{2}{3}$ or if I plug the number in, the answer is bigger than $80.)*
- Does that mean that the values less than $6\frac{2}{3}$ are not solutions to the inequality? **(Challenging)** *(Yes, that's true; they don't work because when you plug them in, the money is less than $80.)*
- I heard you say that we can multiply the number of hours by the amount of money she makes per hour to verify whether a value is a solution to the inequality or not. When we substituted values greater than $6\frac{2}{3}$ into the inequality, a true statement resulted. The values greater than $6\frac{2}{3}$ represent the solutions to the inequality since true statements result. When we substituted values less than $6\frac{2}{3}$, a false statement resulted. This means they are not part of the solution set. In other words, they are NOT solutions to the inequality. **(Revoicing and Recapping)**

LESSON GUIDE 2

EU: Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context.

- How is this situation that involves earning money as a lifeguard different from other contexts that we've worked with? *(In this problem, she wants to earn "more than" a certain amount of money.)*
- Who can add on to this in their own words? *(When we worked with equations, we had x = "some number" as the final answer. In this problem, we have $x = 9$, $x = 10$, etc.)*
- We agreed that this problem has many solutions. Do all of the solutions make sense in the problem situation? In other words, are all values greater than $6\frac{2}{3}$ solutions to this problem? *(Yes, they all make sense because any value greater than $6\frac{2}{3}$ earns more than \$80.)*
- Some of the groups showed us a number line.
- Tell me about the number line. *(It starts here and goes on forever.)*
- Come up and put points on the number line that represent solutions to the inequality. *(Student identifies several points on the number line.)*
- Wait a minute, someone put up 7, 8, 9. There are a whole bunch of numbers between 7 and 8. Do any of these numbers solve the inequality? **(Challenging)** *($7\frac{1}{2}$ sure does. So does $7\frac{3}{4}$, 7, $7\frac{1}{8}$.)* (Have students plot the points on the number line in front of the room.)
- You can see that we are filling in so many points. Mathematicians do this by shading in the line. Why do I have an open circle around the $6\frac{2}{3}$? *(Because it doesn't make the inequality true.)*
- Do the number lines that we put up agree with the solution set showing x greater than $6\frac{2}{3}$?

<u>*Revisiting the claim:*</u>

- We heard the claim that any value greater than $6\frac{2}{3}$ hours earns more than \$80. Who agrees or disagrees with this claim? *(I disagree because she is not going to work any number of hours, such as a billion hours.)*
- I'm hearing some people say that all of the values are solutions, but then other people are saying that not all of the solutions make sense in this problem context. **(Revoicing)**
- Which argument does everybody agree with?
- Can they both be correct? *(Yes, because there are many solutions, but they won't all make sense because we're dealing with hours and dollars.)*
- Who agrees with this statement?
- What number of hours makes sense in this problem? What number of dollars makes sense? *(Hmm…less than 50? Less than 100? How many hours does she plan on working?)*
- Can she work 500 hours? *(I'm not sure; how many hours is she working per week? Per month? How long will she be a lifeguard?)*
- This conversation is a good one. Inequalities have infinitely many solutions. We can see them all on this number line. In this case, we know that any value greater than $6\frac{2}{3}$ hours makes the inequality true, but we can debate which values actually make sense in this problem because we must always consider the context. Maleeka most certainly will not work a million hours over the summer. We would need more information to determine the exact number of hours that she will work as a lifeguard. However, I think we can agree that although an inequality has infinitely many solutions, not all of these values will make sense in the context. **(Marking)**
- What inequality represents this solution set? Is the solution set "greater than" or "greater than or equal to"? How do we know? *(We can say x is greater than $6\frac{2}{3}$. If we are dealing with whole number values, the solution is $x \geq 7$.)*
- I'm hearing us say that an inequality represents this problem situation because all of these solutions (pointing to the number line) make the situation true. We can write this solution set as an inequality, but, again, we have to consider whether these values necessarily make sense in the problem situation. **(Recapping)**

LESSON GUIDE 2

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.

EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- We saw that some students used an equation to solve this problem. I noticed that one of the groups calculated and set up an inequality.
- I noticed one group calculated the number of hours by setting up and solving an inequality to determine the number of hours Maleeka worked to make $750. Can you explain your method? *(We set up and solved* $12x > 750$*.)*
- Wait a minute, before we get to the solution, who can restate in his/her own words what this inequality represents in this problem? *(We know she makes $12 per hour and we're looking for the number of hours that make her pay greater than $750.)*
- Who can say back in their own words? *(12 * x hours is greater than $750.)*
- Who understood what this group said and can say it in their own words? *(If you multiply $12 per hour by x hours, you have to get more than $750.)*
- 12 times x hours is the amount earned. $750 is the amount of money earned. This time we want the $12x$ to be bigger than $750. **(Marking and Revoicing)**
- Let's look at how this group determined their solution. *(We divided both sides by $12 per hour.)*
- Can we just do that, divide both sides? If so, why or why not? **(Challenging)**
- Let's try this hypothesis out with numbers and see! We know for sure that $24 > 12$. What happens when we use the properties of equality and divide both sides by 12? *(We get* $2 > 1$*.)*
- Is that true or false? *(True.)*
- Now let's try $48 > 24$ and divide by 12. What happens? *(We get a true statement,* $4 > 2$*.)*
- Let's try a few more examples. Choose an inequality, and multiply or divide both sides by the same positive number. Try values that are not 12. Also consider numbers less than one. Make some observations. Are all of the statements still true after multiplying or dividing by the same positive number, or are they sometimes false? **(Challenging)**
- What do you notice? *(Whenever we divide or multiply the inequality by a positive number, the new statement is still true.)*
- Who can add on? *(All of the examples that we used worked.)*
- Who can say why? *(If a number is less than another number, and you multiply both numbers by the same positive value, the smaller number will still be smaller.)*
- Can somebody else add on to this using a real-world example? *(If I have $10 and she has $8, if we cut both values in half then I still have more money. I have 5 and she has 4. Same as if we doubled or tripled our amounts.)*
- So let's mark this. When we divide or multiply an inequality by a positive number, the truth of the inequality remains. In other words, if one number is larger than another number, and you divide or multiply each value by the same positive number, the resulting inequality is still a true statement. **(Recapping)**

LESSON GUIDE 2

Application	Suppose Maleeka's long-term goal was to make more than $1000. Determine the number of hours that Maleeka must work to meet this goal. Represent your solution set using words, an inequality, and a number line.
Summary	See above in the recapping statement.
Quick Write	Do all of the solutions to the inequality in question 2 make sense in this problem situation? Explain why or why not using words and specific examples.

TASK
3

Name____

Video Game Rental

Sore Thumb, Inc. is an online gaming company. They are exploring pricing options. Sore Thumb, Inc. has found that most gamers wish to spend less than $12.50 per month for an online gaming subscription.

One pricing option is to charge customers a membership fee of $2.79 each month and then $0.23 for every hour spent playing their games online over the course of the month.

1. Use an inequality to determine the number of hours that gamers may play and spend less than $12.50 per month. Show all work and explain your reasoning.

2. A second pricing option is shown in the table below:

Playing Time (hours)	Total Cost (dollars)
0	4.00
1	4.20
2	4.40
3	4.60
4	4.80

Determine the number of hours that gamers will be able to play if they wish to spend less than $12.50. Show all work and explain your reasoning.

Video Game Rental

LESSON GUIDE 3

Rationale for Lesson: In this lesson, students are given a context and specifically asked to solve the problem situation algebraically using inequality notation. Students then compare and contrast two inequalities.

Task 3: Video Game Rental

Sore Thumb, Inc. is an online gaming company. They are exploring various pricing options. Sore Thumb, Inc. has found that most gamers wish to spend less than $12.50 per month for an online gaming subscription. One pricing option is to charge customers a membership fee of $2.79 each month and then $0.23 for every hour spent playing their games online over the course of the month.

1. Use an inequality to determine the number of hours that gamers may play and spend less than $12.50 per month. Show all work and explain your reasoning.
2. A second pricing option is shown in the table below:
   Determine the number of hours that gamers will be able to play if they wish to spend less than $12.50. Show all work and explain your reasoning.

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure.	

LESSON GUIDE 3

Essential Understandings	• An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true. • Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.
Materials Needed	• Student reproducible task sheet • Graph paper (as needed for students approaching the problem graphically) • Calculator

SET-UP PHASE

Read along silently while someone reads the task aloud. Does anybody play video games? How is this company, Sore Thumb, Inc., pricing its membership? What is a one-time charge and why might a company charge this sort of fee? Do you think this would be a successful model? Take about ten minutes to work on this problem individually before sharing with a partner. The directions say to solve this problem algebraically by setting up and solving an inequality. You may use other methods at first to make sense of the problem.

EXPLORE PHASE

<table>
<tr><th>Possible Student Pathways</th><th>Assessing Questions</th><th>Advancing Questions</th></tr>
<tr><td>Uses a numeric strategy (to test to see if they satisfy the inequality).

For example:
0.23 * 2 + 2.79 = 3.25</td><td>Can you explain why you multiplied 0.23 by this value and why you added 2.79 (point to the student's values)? What does each expression mean in terms of the problem context?</td><td>How can you determine the cost for 4 hours? 5 hours? What pattern(s) do you notice?</td></tr>
<tr><td>Creates a table of values.

<table><tr><th>Playing Time (hours)</th><th>Total Cost (dollars)</th></tr><tr><td>0</td><td>2.79</td></tr><tr><td>1</td><td>3.02</td></tr><tr><td>2</td><td>3.25</td></tr><tr><td>3</td><td>3.48</td></tr></table></td><td>How did you set up the table? What do the values represent?</td><td>How will you use this table to determine the number of hours that somebody may play video games and stay under the indicated dollar amount?</td></tr>
<tr><td>Sets up and solves an equation.

$0.23x + 2.79 = 12.50$</td><td>Tell me about your equation. How did you solve the equation?</td><td>How many solutions does the problem have? How can you use your equation to determine all of the solutions?</td></tr>
<tr><td>Sets up and solves an inequality.

$0.23x + 2.79 < 12.50$</td><td>What does each term in your inequality represent? How did you solve the inequality?</td><td>How can you verify your solutions in terms of the inequality? What about in terms of the context?</td></tr>
<tr><td>Group finishes early.</td><td>Can you tell me about your solution strategy and how you verified that your solutions solve the inequality? How do your solutions relate to the problem context of video game rentals?</td><td>Analyze the table in Question 2. How can you set up and solve an inequality to determine the playing time for this pricing option?</td></tr>
</table>

LESSON GUIDE 3

SHARE, DISCUSS, AND ANALYZE PHASE

EU: An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true.
EU: Some real-world situations have the potential of infinitely many solutions and are therefore more appropriately modeled by inequalities than equations (e.g., situations that involve language such as "at least," "no more than," "fewer than," etc.). However, not all of the values will make sense in the context.

- A few groups tried setting up an equation for this task. Please come up and explain your thinking. *(We wrote the equation $0.23x + 2.79 = 12.50$.)*
- Does everyone agree with them that this represents the situation? *(Yes, no.)* Why not? *(The problem said to use an inequality, not an equation. They are different.)*
- What about the problem suggests that an inequality is more appropriate than an equation? *(Well, it said in the problem "spend less than." That can mean any number less than 12.50.)*
- Who can explain what this group just said about the difference between an equation and an inequality? *(They said that there are lots of answers to this and in an equation there is only one answer.)*
- I'm hearing from some groups that the words "less than" is an indicator. I'm also hearing groups say that there will be many solutions to this problem. These are key words that many people use to make sense of these types of problems. Previously, we had problems with "more than." In this problem, we are asked about "less than." **(Marking)**
- Tell us about how you used an inequality to make sense of this problem situation. What inequality did you write? *(We wrote that the amount spent had to be less than \$12.50.)*
- Can someone come write that inequality on chart paper? *(Writes: $0.23x + 2.79 < 12.50$)*
- What do each of the terms in the inequality represent?
- Before we discuss the solution set, or the various solutions to this problem, who can tell me what it means to be a solution to this inequality? *(When you substitute a value in for the variable it makes a true statement. The output will be less than \$12.50.)*
- Can someone say that again in their own words? *(When you put a value in for x, then you multiply by 0.23 and add 2.79, it comes out to less than 12.50.)*
- Can someone say that back referring to the context of the problem? *(When you choose a number of hours and try multiplying it by 23 cents per hour, then add the fee of \$2.79, it comes out to less than \$12.50.)*
- So the amount of money spent on the hourly rate plus the fee still comes out to less than \$12.50, so we cannot put in the equal sign because $0.23x + 2.79$ is not the same as 12.50. **(Revoicing)**
- How many solutions will there be to this inequality? *(Lots of numbers will work.)*
- What numbers will work? Will non-whole numbers work? Let's try a few–try substituting in 6, -20, $\frac{1}{2}$.
- It sounds like we're picking up on the fact that inequalities have infinitely many solutions that you can substitute in order to make a true statement, but that we need to make sure the solutions make sense. **(Recapping)**
- Will every number in our solution set make sense in the context of the problem? **(Challenging)** *(No, only numbers that represent the amount of hours, so probably whole numbers.)*
- What inequality did you write for the second pricing option? *(We wrote the number of hours times 0.20 plus \$4 has to be less than \$12.50.)*

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.

EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- We have talked about why the inequality is the better representation of this task. However, did the equation that the first group wrote actually help us to solve the problem? *(Yes.)*
- How is that the case? *(We determined the number of hours that will cost exactly $12.50. Then we knew the gamers could play for less than that amount.)*
- Someone say that in their own words. *(When we solved the equation we knew that the answer showed how many hours it was for exactly $12.50. Any time less than that answer will cost less than $12.50.)*
- How can we represent what was just said about the solutions on a number line? *(It has to be less than the number of hours that equaled to $12.50.)*
- What was that number that is the breaking point between equaling $12.50 and more than $12.50? *(It was 42.2 when we solved the equation.)*
- Who can say in their own words how they used the solution to the equality to solve the inequality and then add on by explaining how the solution to the equation divides the number line? *(The solution to the equation divides the number line into all values less than 42.2 and then all numbers greater than 42.2.)*
- So the solution to the equation divides the number line and then you can use this to determine all the solutions to the inequality. **(Marking)**
- Then we can set up and solve an inequality? *(Yes, it was like solving the equation, but then we said any amount of hours less than 42.2 worked as well.)*
- Who can explain what their solution $x < 42.2$ means in this problem? Explain in words and then on the number line. *(42.2 was the most hours you could be gaming and hit $12.50. Everything less than that costs less. So, on the number line shade up to 42.2.)*
- How does 42.2 divide the number line and help us make sense of the solution set?
- How was this process the same/different than our work with equations? **(Challenging)**

EU: Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers.

- Does 42.2 count as an answer? *(No, because at 42.2 hours, the amount of money EQUALS $12.50. We want it to be less than $12.50.)*
- If we try 42 hours, will it work? Why?
- If we try 43 hours, will it work? Why not?
- Looking at our solution set, we touched earlier on how we can use the inequality to test our solution set. Does the number line that we drew support the answers we got for 42 and for 43? *(Yes, 42 hours was less than $12.50 and 43 hours was over that amount.)*
- Who can choose a different value in our solution set and show whether it makes a true statement?
- Can we choose $x = -2$? Does this work? **(Challenging)** Why or why not?
- How does the solution set differ from the first pricing option to the second pricing option? **(Challenging)**

LESSON GUIDE 3

Application	Create a company with a different one-time fee and hourly rate. Determine the number of hours that gamers may play in order to pay less than $12.50 per month. Choose a point in your solution set and verify that it makes a true statement within the inequality.
Summary	Explain the steps involved in solving an inequality such as $5x - 1 < 9$. Then explain how to test a value within your solution set. Let's compare to see if everyone used the same steps.
Quick Write	Mary makes the following claim: "An inequality always has an infinite number of solutions." Leonard claims: "Inequalities may have infinitely many solutions. It depends on the problem situation." Explain how each student may be correct. Use our work in the Video Game Rental task to support your reasoning.

Support for students who are English learners (EL):

1. Create a running list comparing properties of equations and inequalities so that the similarities and differences are more evident to students who are English learners. Pointing to the solution set, the breaking point on the number line, the greater than and less than symbols, etc. will also help students who are English learners develop mathematical vocabulary.

TASK 4

Name____

The Possibilities are Endless!

1. Determine whether each of the possible values provided in the box below is a solution to

$$2x - 7 = 15,$$
$$2x - 7 > 15, \text{ or}$$
$$2x - 7 < 15.$$

Record the solutions in the table of values.

Possible Solution Values

11.01	11	12	0	8
14	10.98765	-7000	12	5
-5	105	$\frac{1}{2}$		

$2x - 7 = 15$	$2x - 7 > 15$	$2x - 7 < 15$

Explain your solution strategy.

TASK 4

2. Represent the solution to each equation/inequality on the number lines below, and then describe any patterns you notice.

The Possibilities are Endless!

LESSON GUIDE 4

Rationale for Lesson: Students solidify their strategies and representations for inequalities.

Task 4: The Possibilities are Endless!
1. Determine whether each of the possible values provided in the box below is a solution to $2x - 7 = 15$, $2x - 7 > 15$, or $2x - 7 < 15$. Record the solutions in the table of values. Explain your solution strategy.
2. Represent the solution to each equation/inequality on the number lines below, and then describe any patterns you notice.

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Essential Understandings	• Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.	
Materials Needed	• Student reproducible task sheet • Calculators (optional)	

LESSON GUIDE 4

SET-UP PHASE

Read along silently as someone reads the task aloud. Who can summarize in their own words, without giving away a solution strategy, what is being asked of you in this task? What equations/inequalities are you using in this task? What do the values in the box represent? We've learned a lot about inequalities in the first three tasks. Use what you have learned about how to solve inequalities, and what it means to be a solution to an inequality, to come up with an efficient method for figuring out whether these values are solutions or not to each equation/inequality given.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Substitutes each value into the equation/ inequalities.	Explain your method to me. How do you know whether each value is a solution to the equation/inequality?	How can you use the solution to the equation to simplify your work on the inequalities?
Chooses one value and uses this value as a benchmark for determining other solutions. For example, since 2(5) – 7 is less than 15, then substituting every other value less than 5 must also result in a value less than 15.	Can you explain your method to me? How did you choose your value(s)? How many solutions does the equation have? How many solutions does each of the inequalities have?	What is the solution to the equation? How does the solution to the equation divide the number line?
Solves the equation $2x - 7 = 15$ and uses this solution as a benchmark for determining the other values.	Can you explain your method to me? How did you use the solution to the equation to help you solve the inequalities?	How do you know that every value to the right of $x = 11$ is true for $2x - 7 > 15$?
Answers all parts of the task correctly.	How did you determine what to draw on each number line?	Suppose I changed the equations/inequalities to $-2x - 7 = 15$, $-2x - 7 > 15$, and $-2x - 7 < 15$. How does this change where you place the possible solutions on the number line?

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE 4

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.

EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Let's start with the equation. Will the group that solved that task algebraically come share how they determined the solution to the equation? *(We added 7 to both sides and then divided by 2.)*
- Did anyone follow their thinking about how they solved the problem? Someone say back the steps that they took to solve the equation.
- Why did they add 7 to both sides? *(If we add something to both sides of an equation, then it still stays equal.)*
- Did everyone hear that? The students said that because they added the 7 to both sides, they did not change the balance of the equation. **(Revoicing)**
- Does that thinking apply to all four operations? **(Challenging)** *(Yes, because next we divided each side by 2 and the balance stayed the same, so it works for at least addition and division.)*
- So if we apply the properties of equality, they can help us solve an equation. **(Marking)**
- How does this solution look on the number line? *(The solution is 11. We put a point there because it is the only answer.)*
- Which group solved the inequalities algebraically? Come show us what you did. *(We still added 7 and divided by 2.)*
- So the same properties of equality were used? *(Yes. Adding, subtracting, dividing, or multiplying both sides preserves the relationship. If the two expressions are equal, or if one is greater than the other, this doesn't change when you do something to both sides.)*
- So what answers did you come up with to the inequalities? *($x < 11$ and $x > 11$)*
- Which answer goes with which inequality?
- Does everyone agree?
- How do the answers to the inequalities relate to the answer for the equation? *(The equation and inequalities are the same except the one inequality, $x > 11$, represents values that make $2x - 7$ greater than 15; the other, $x < 11$, represents values that make $2x - 7$ less than 15; and then the equation represents the values which are equal to 15.)*
- What do the answers to the inequalities look like on a number line? Come show us your number line. *(We marked 11 and then shaded everything less than 11 for the less than inequality.)*
- Did others come up with the same number line?
- Did you think that 11 is part of your solution to the inequality? *(No, but it is everything less than 11.)*
- Why don't we start at 10 then? *(Some students reply that it would work to start the number line at 10.)*
- Does everyone agree? No? Why not? *(Because there are still numbers that are between 10 and 11.)*

LESSON GUIDE 4

EU: Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers.

- Some students substituted the values into the equation instead of solving the equation algebraically. What do we mean when we say, "substituting a value into the equation"? *(You plug in the value; in this case, you multiply by 2 and subtract 7, then you see if it equals 15.)*
- Who can restate what she just said about plugging in the values? *(You pick a value, put it in for x, and then follow what the equation tells you to do.)*
- Will every value you choose and substitute in the problem work for the equation $2x - 7 = 15$? **(Challenging)** *(No, the answer is 11.)*
- How many solutions exist for the equation? *(There's only one solution for the equation.)* How do you know that? *(Well, it has to end up being equal to only 15; that's one value so only one value will make that true.)*
- I hear you saying that the equation only has one solution. **(Marking)**
- Do all equations only have one solution? Agree or disagree? **(Challenging)**
- What about the inequalities; how many of the answer choices worked for each of them? *(There were a couple answer choices that worked for each.)*
- Does everyone agree that there are multiple answers for each inequality?
- Why are there lots of answers to the inequalities but only one for the equation? *(The inequalities say that the answer is any number less than 15 or greater than 15–lots of answers make that true.)*
- Where do we see that on the number line the other group showed? *(They shaded everything less than 11 and it hit EVERY number less than 11, even to negative infinity!)*
- So I hear you saying that negative numbers will work in the $2x - 7 < 15$ inequality. How can we prove that is true? *(We can pick a negative number and substitute it for x.)*
- Let's all try a negative number to see if what he said is correct.
- Is it possible for a solution to be in two of the categories? **(Challenging)** Why or why not?

Application	No application.
Summary	Who can explain the key ideas that we learned about inequalities over the course of the last four lessons?
Quick Write	How can knowing the solution to $5x - 3 = 27$ help you know the solution to $5x - 3 < 27$?

TASK
5

Name____

Deep Dark Secret

The deep sea is the lowest level of the ocean floor. Sunlight does not reach the deep sea, and scientists are discovering that it is home to some amazing creatures that live in total darkness. Deep sea creatures live more than 1000 feet below the water's surface.

Marine biologists in an underwater vessel are descending to study a new species of fish that was discovered in the deep sea. Their vessel is currently located 100 feet below the surface (-100 feet). From this location, the biologists start their timer at t = 0 and begin their descent. They descend at a rate of 25 feet per minute. The vessel continues descending at this constant rate until it reaches the deep sea, at which time it will stop to study the sea creatures.

a. Write an inequality to represent the depth of the vessel at any point in time, t, after the vessel has reached the deep sea, where t represents the number of minutes since the vessel has left its original position of -100 feet.

b. After how many minutes will the vessel be within the deep sea level? Explain how you know.

c. Graph your solution to part b on the number line.

LESSON GUIDE 5

Deep Dark Secret

Rationale for Lesson: In the previous lesson, students used algebraic strategies to solve inequalities in various representations. In this task, students are presented with a context that can be solved in a variety of ways, but because of the negative rate of change, their currently known algebraic strategies do not work. (In the next three lessons, students will continue exploring why it is necessary to switch the inequality symbol when multiplying or dividing by a negative number.)

Task 5: Deep Dark Secret

The deep sea is the lowest level of the ocean floor. Sunlight does not reach the deep sea, and scientists are discovering that it is home to some amazing creatures that live in total darkness. Deep sea creatures live more than 1000 feet below the water's surface.

Marine biologists in an underwater vessel are descending to study a new species of fish that was discovered in the deep sea. Their vessel is currently located 100 feet below the surface (-100 feet). From this location, the biologists start their timer at t = 0 and begin their descent. They descend at a rate of 25 feet per minute. The vessel continues descending at this constant rate until it reaches the deep sea, at which time it will stop to study the sea creatures.

a. Write an inequality to represent the depth of the vessel at any point in time, t, after the vessel has reached the deep sea, where t represents the number of minutes since the vessel has left its original position of -100 feet.

b. After how many minutes will the vessel be within the deep sea level? Explain how you know.

c. Graph your solution to part b on the number line.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure.	

Essential Understandings	• An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true. • Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.
Materials Needed	• Student reproducible task sheet • Graph paper, number lines, calculators (optional)

LESSON GUIDE 5

SET-UP PHASE

Read the task quietly on your own and then I would like a volunteer to read the task aloud. What do we know about the deep sea and scuba vessels? Take a look at these pictures of deep sea creatures that I found. How do you think these creatures may be the same/different than those that live within reach of sunlight? New species are still being discovered and, as you can see, some of them look pretty different. What do you know about the path of this underwater vessel? What does it mean to descend? How fast is it descending? As you work, you have graph paper, number lines, and calculators to use as you see fit to come to a solution.

EXPLORE PHASE

<table>
<tr><th>Possible Student Pathways</th><th>Assessing Questions</th><th>Advancing Questions</th></tr>
<tr><td>Can't get started.</td><td>Can you explain what the problem is about? Where is the vessel at the start of the problem?</td><td>Where is the vessel after one minute? Two minutes? What is the pattern?</td></tr>
<tr><td>Creates a table of values.
<table><tr><th>Time (min)</th><th>Depth (feet)</th></tr><tr><td>0</td><td>-100</td></tr><tr><td>1</td><td>-125</td></tr><tr><td>2</td><td>-150</td></tr><tr><td>3</td><td>-175</td></tr></table></td><td>How did you create the table of values?</td><td>What patterns do you see in the table? How can you use the patterns to determine when the depth is -1000 feet?</td></tr>
<tr><td>Uses a numeric guess-and-check method.

For example, after 10 minutes, the vessel has dropped 250 feet to 350 feet below the surface. After 20 minutes, the vessel has dropped 500 feet to 600 feet below the surface.</td><td>Can you explain your method? How are you determining the depth of the vessel over time?</td><td>What pattern do you notice? How can you use this pattern to come up with an algebraic expression?</td></tr>
<tr><td>Sets up and solves an equation.

$-100 - 25t = -1000$
$-25t = -900$
$t = 36$</td><td>How did you set up an equation to represent this problem?
What does each term represent in this equation?</td><td>What does the solution to the equation mean in the context of the problem? How many solutions are there to this problem situation? How do you know?</td></tr>
<tr><td>Sets up and solves an inequality incorrectly.

$-100 - 25t < -1000$
$-25t < -900$
$t < 36$</td><td>How does your inequality represent this problem situation? What does your solution represent in this problem situation?</td><td>$t = 1$ is in the solution set to $t < 36$. Will the vessel be in the deep sea at that time? What does this suggest about your solution set?</td></tr>
</table>

SHARE, DISCUSS, AND ANALYZE PHASE

EU: An inequality is a statement comparing the relative magnitude of two expressions. As a result, it can be judged true or false. The solution set of an inequality contains all of the values of the variable that make the statement true.

Use of a diagram:

- As I circulated, I noticed several different solution strategies. Let's look at this group's diagram and calculations. How did you solve the problem? Please make sure to explain your diagram. *(We drew the surface of the ocean, and a number line pointing down below it to the deep sea. We marked -100, because that's where they started. Then we marked -1000, because that's the deep sea. When we looked at the diagram, we realized that there's 900 feet between 100 feet below the surface and 1000 feet below the surface. So we divided by 25 and got 36.)*
- Who understands what this group did and can explain the method in your own words?
- But wait–where on the diagram can we see the 900 feet? And why do we divide by 25? What does the 25 represent?
- Is "36 minutes" the answer to the problem? *(No, there's other numbers that work, too.)*
- Who can use the diagram this group drew to help us understand why there are other solutions? **(Challenging)** *(Look here. On the diagram, if the boat goes below -1000, it's in the deep sea. But at 36 minutes, it just hit the deep sea. So after another minute and another minute, like 37 and 38, it will get lower and lower.)*

Use of the table:

- Group B, you used a table to solve the problem. Please explain how the table helped you think about the problem.
- Who can explain this group's methods and then explain how they know their solutions "work" for this problem situation? *(They created a table of values and continued the pattern in the table until they found the values that were more than 1000 feet below the surface. They found that this was true for lots of values.)*
- Who can be more specific about the "lots of values"? *(The time values greater than 36 minutes all work.)*
- How does this solution to the problem compare to the first group's solution?
- Who can restate this method in their own words and then add to it by representing the solution set with an inequality and on the number line? *(They basically determined all of the time values that make the depth lower than 1000 feet by extending the table until the depth was deeper than -1000 feet. I can represent this solution as $t > 36$ or by shading on the number line all the numbers larger than 36.)*
- So I'm hearing that the solution set is represented by an inequality because we're looking for all of the time values for when the vessel is more than 1000 feet below the sea level. **(Revoicing)**
- Let's look at the inequality $t > 36$. An inequality is a comparison between two numbers. We saw that before in the Lifeguard task, for example; in this task, we are comparing t and 36. **(Marking)**
- How is that comparison represented on the number line?
- What does the shading above 36 say about the solution to the problem? *(It's the numbers bigger than 36. It's 36, 37, 38 minutes and we keep going.)*
- But we shaded the space between 36 and 37. Why? **(Challenging)** *(Oh, it's fractions, too. 36.5 minutes also works. It's also in the solution set.)*
- So I'm hearing you say that an inequality has infinitely many solutions, and that all of the values on your number line represented by the inequality $t > 36$ are solutions. **(Marking and Revoicing)** So let's take a look at another way to solve the problem–using an inequality to represent the problem.

LESSON GUIDE 5

EU: Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form *a* > *b* (or *a* ≥ *b*) where a and b are real numbers.

- Group C, please explain how you represented this problem with an inequality. *(We set up the inequality -100 – 25t < -1000.)*

Use of an algebraic expression:

- Who can say what the expression -100 – 25*t* represents within the context of the problem? *(The depth of the vessel at any minute after it starts descending.)*
- Let's explore why it represents the depth. What do each of the terms in -100 – 25*t* represent in the context of this problem situation? **(Challenging)** *(The vessel starts 100 feet below the surface, so -100; it descends at a rate of 25 feet per minute, so -25; and t is the number of minutes.)*
- Why multiply -25 times *t*? What does that represent? *(That's how many more they went down.)*
- Who understood that and can say it in your own words? *(If they go down 25 feet per minute, after 1 minute, they're down 25; after 2, 50; after 3, 75. So you multiply to get those numbers.)*
- So you are saying that we multiply the number of minutes, t, by the rate of descent, -25 feet per minute, to determine the total number of feet that the vessel has descended after *t* minutes. **(Revoicing)**
- So now I'm wondering about the -1000. What does it represent in the context of this problem situation?
- Let's look at the whole inequality again. -100 – 25*t* < -1000. I heard you say that -100 – 25*t* represents the depth of the vessel at any time, t, and -1000 represents the depth of the vessel once it reaches the deep sea. What does the "less than" symbol say? And why "less than" and not "greater than"? **(Challenging)**
- Somebody in another class represented the inequality with a greater than symbol because we were looking for times that the vessel is MORE than 1000 feet below the surface. Do you agree or disagree with this inequality? Explain. *(No, I don't agree because the numbers are getting farther away from 0 in the negative direction, which actually means they are getting smaller.)*
- So we are still comparing two numbers in this inequality, -100 – 25*t* < -1000. Can someone read this inequality in words, rather than in symbols? What comparison does it make? *(It says, the depth of the boat started 100 feet below the surface and kept going lower by 25 feet per minute for t minutes, and we want to know when that depth will be lower than 1000 feet below sea level. It's comparing the depth of the vessel to -1000 feet.)*
- Very nice. You remembered that an equation/inequality is very much a question–for what numbers will the equation or inequality be a true statement? So let's check some numbers out to see if we have the correct inequality. **(Marking)**
- We know, for example, that *t* = 40 is a solution to the problem from Group A's and B's work. How do we know if *t* = 40 is a solution to this inequality? *(40 [-25] = -1000 and the -1000 + [-100] = -1100. Then -1100 < -1000 is a true statement.)*
- What about the other values in the table that are solutions? Do they also make the inequality a true statement? Take a minute to try a few.
- Again, looking at this table of values from earlier, if I substitute these values (pointing to values less than 36) into the inequality, what will I know? *(If I plug in 5, then 5 [-25] – 100 < -1000 simplifies to -225 < -1000, which is a false statement. -225 is to the right of -1000 on the number line. It results in a statement that is false.)*

- Are there other values for which the inequality is false? Are they solutions to the problem or not?
- So we substituted several values into the inequality. Some made the inequality a true statement, while others made the inequality a false statement. Do we have to substitute all values, or can we say, at this point, which make the inequality a true statement? Explain. *(Yes. 36 divides the number line. All values greater than 36 are true, all values less than 36 are false.)*
- Somebody say more about this. What did this group mean when they said "36 divides the number line"? *(They basically solved the equation. 36 solves the equation $-100 - 25t = -1000$ and this solution divides the number line into values greater than 36 or less than 36.)*
- Who can add on? *(The values less than 36 make the inequality false, and the values greater than 36 make the inequality true.) (Yes, and so the values greater than 36 are solutions and the others are not.)*
- Interesting. So the solution to the equation divides the number line into two sets, only one of which contains solutions to the inequality. By testing using substitution, we can see which set makes the inequality a true statement, and so which set represents the solutions to the problem. **(Revoicing and Recapping)**

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.
EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Now that we've explored a couple of different approaches, we know that the solution set is all numbers greater than 36, or $t > 36$. Who can confirm this algebraically by solving the inequality?
  $-100 - 25t < -1000$
  $-25t < -900$
  $t < 36$
- Do you agree or disagree? Explain. *(I disagree because we substituted values into the inequality and we know that $t > 36$ is the solution set.)*
- So I'm hearing you say that this solution is incorrect, because we substituted values less than 36 into the inequality and problem situation and we know that they are not solutions; that the vessel is not in the deep sea before 36 minutes. **(Marking)**
- So who can check our steps to see if the arithmetic is correct and whether we used the properties of equality correctly?
- I'm hearing you say that we solved the inequality correctly, but solving the inequality led to the incorrect answer in this case. Before we give up on an algebraic method, let's think about what is different about this problem. In Tasks 1-4, our previous inequality tasks, the algebraic method worked. Now it failed us. What is different that we need to consider? *(We still have an inequality. What's different is that we're dealing with negative numbers.)*
- You noted a couple of important ideas; first, that the situation can be represented by an inequality and second, that the situation contains negative numbers. **(Marking)**

continued on next page

LESSON GUIDE 5

- Let's consider, then, where the logic fell apart in the solution steps. Let's check each inequality to see if it is equivalent to the one above it. We know that $-100 - 25t < -1000$ is true when $t > 36$, because we just checked. What about $-25t < -900$? Check out some values. Let me know what works.
- So the first two inequalities are equivalent because they both have the same solution set. **(Marking)**
- The only step that has a different solution set is the last one. Remind us…what did we do in that step? *(We divided by -25. Maybe that's the problem.)*
- Let's try this hypothesis out with numbers and see! We know for sure that $100 > 75$. What happens when we use the properties of equality and divide both sides by, first, 25? (We get $4 > 3$.)
- Is the solution true or false? *(True.)*
- Now let's try $100 > 75$, but divide both sides of the inequality by -25. What happens? *(We get a false statement! -4 is NOT greater than -3. In fact, $-4 < -3$.)*
- Let's try a few more examples. Choose an inequality, and multiply or divide both sides by the same positive number, then by the same negative number. Make some observations about which results are true and which are false and consider…why are some statements true and others false? **(Challenging)** *(Whenever we divide or multiply the inequality by a negative, we get a false statement.)*
- Who can add on? *(So it turns out the reverse comparison is true.)*
- Why? Who can say why? *(Because we get opposites.)*
- Who agrees and can add on or explain in your own words? *(The numbers change to the other side of the number line.) (Because dividing by a negative gets opposites, the numbers switch on the number line. When we are on the other side of the number line, size works backwards. So > becomes < and < becomes >.)*
- So let's mark this. When we divide an inequality by a negative number, the numbers on both sides of the inequality are negated. Negating a number moves it, by the definition of negating, to the opposite side of the number line. The number farther from zero is bigger on the positive side of the number line, but smaller on the negative side of the number line. So negating both numbers reverses the ordering and comparison of the two numbers. **(Recapping)**
- Let me repeat that, but let's connect the idea to a number line, (Student name), will you point out on the number line what is happening? Let's start with a true statement, $6 > 2$. Show us where those numbers are on the number line. (Second student name), will you write the inequality symbolically?
- Now let's divide by a negative number, say -2. Show us the resulting number's inequality; now point to those numbers on the number line. Now it's the -3 that is farther away from zero, and so is smaller than the -1.
- Since an inequality is a comparison, and we want to keep the inequality a true statement, in order to keep that comparison true, what must we do?
- So I think I'm hearing you say that if we switch the inequality symbol, we maintain the truth of the inequality. **(Marking)**
- Will this always be the case? When you solve any inequality with a negative coefficient, do you think we will always have to switch the inequality to get the correct answer? **(Challenging)**
- So this looks like something we will need to explore further.

LESSON GUIDE 5

Application	Use two different representations to explain why $t = 50$ is a solution to this problem.
Summary	How was this problem the same/different than other problems that we've studied?
Quick Write	Explain how to test whether a value is a solution to an inequality.

TASK
6

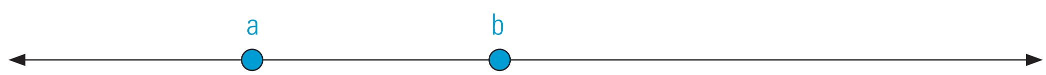
Name____

Josie is so Negative These Days!

Part 1

Josie says, “I’m thinking of two numbers, *a* and *b*. I plotted them on the number line so you can see $a < b$. But guess what happens when I multiply both numbers by -1? They switch! The numbers become $-a$ and $-b$, and the correct inequality becomes $-a > -b$!”

1. Do you agree or disagree with Josie? Support your answer mathematically.



2. If Josie multiplies both sides of $a < b$ by *any negative number*, is it still true that the inequality symbol must be switched? Explain your reasoning.

TASK 6

Part 2 (Extension)

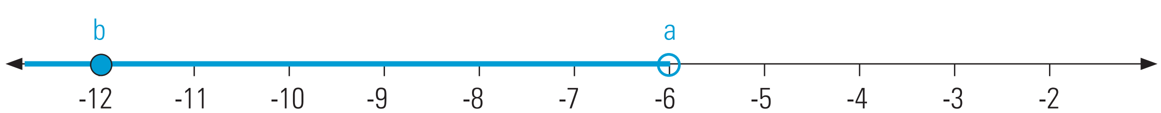
Next, Josie tells a group of friends, "I'm thinking of a number. Add 3 to it. Now multiply by -2. My number is less than 6. See if you can write down all of the possibilities for my number."

Her friends sketch their answers, which are shown below. Determine which responses, if any, are correct. Support your reasoning mathematically.

Darlene: "I sketched my answer on the number line below."

Jonah: "My answer is every integer less than -6."

Caroline: "I also sketched my answer on a number line."



Mica: "My answer is (-5, -4, -3, -2…)"

LESSON GUIDE 6

Josie is so Negative These Days!

Rationale for Lesson: In the last task, students encountered an inequality problem situation involving a negative slope. Students determined the correct answer, but an algebraic solution method did not work, prompting the need to analyze this situation further. In this lesson, students look at numbers on the number line to make sense of WHY multiplying or dividing by a negative number "switches" the inequality.

Task 6: Josie is so Negative These days!
Part 1
Josie says, "I'm thinking of two numbers, a and b. I plotted them on the number line so you can see $a < b$. But guess what happens when I multiply both numbers by -1? They switch! The numbers become $-a$ and $-b$, and the correct inequality becomes $-a > -b$!"

1. Do you agree or disagree with Josie? Support your answer mathematically.
2. If Josie multiplies both sides of $a < b$ by *any negative number*, is it still true that the inequality must be switched? Explain your reasoning.

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	

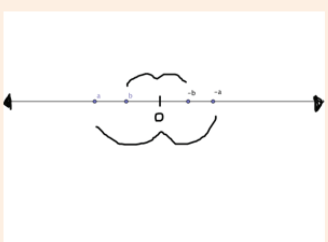
Essential Understandings	• Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.
Materials Needed	• Student reproducible task sheet • Additional number lines, calculators

LESSON GUIDE 6

SET-UP PHASE

Please read the task silently before we read it aloud. Without giving away answers or a strategy, who can explain in their own words Josie's claim? How can you tell from looking at the number line on the task sheet that *a* is less than *b*? What does it mean to support your answer mathematically? *(Student answers.)* Whether you agree or disagree, support your reasoning with evidence and use mathematics to support your argument.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	Can you explain in your own words what Josie is claiming?	What are some possible values for *a* and *b* such that *a* is less than *b*? After multiplying by -1, which value is greater? Explain and then try other values.
Substitutes specific values. Since I know 5 < 6, then is -5 > -6? Yes. I know -4 < 2, so is 4 > -2? Yes.	Can you explain your strategy to me?	It looks as though this "works" for all of the values you are choosing. Can you use the number line to explain why multiplying by -1 reverses the inequality?
Uses opposites and distance from zero to make sense of the problem. 	Can you explain your strategy? How did you determine the location of the opposite values and how did you know this "switched" the inequality?	How do you know this works for all cases? For example, does it work when both numbers are positive? One positive/one negative? Explain.
Finishes early.	Can you explain your solution strategy?	Can you set up and solve an inequality in the extension problem? Which student(s)' thinking is correct and why?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers.

- Group A, tell us how your group made sense of this problem. *(We chose some values for a and b, and multiplied both by negative values. We could see that the values kept switching.)*
- Who can say in your own words how this group used specific values to test whether Josie's statement is correct? *(They substituted values such as $-4 < -3$ and $6 < 8$ to test what happened to both values when they multiplied both sides of the inequality by a negative.)*
- This group said that they know $a < b$. It's mentioned in the problem, but how do we know a is less than b from the number line? *(We know a is less than b because it's to the left of b on the number line.)*
- Does it matter whether the values that they substitute for a and b are positive or negative? *(No, all that matters is that a is smaller than b.)*
- So I'm hearing that what matters when we substitute values for a and b is that we keep the inequality $a < b$ a true statement. **(Marking)**
- Group A mentioned that the values kept switching. What does this mean? *(After we multiplied by -1, the new values switched to the other side of the number line.)*
- How did the new values then compare to one another? How did you make the comparison? *(The new values didn't stay in the same order. They switched.)*
- Can someone add on to that? *(The number that was "less than" switched. If a value is to the right of another value on the number line, it is greater. The number that was less than switched to the right of the other number. It became the greater than number.)*
- I'm hearing you say that we can choose values for a and b, and test them to make sure that the original inequality, $a < b$, is true. Then we can test what happens when we multiply both sides of the inequality by -1. This is important, because it's a strategy that scientists and mathematicians use all of the time. They test specific values to see if they work and then start to come up with a reason why. This is what we are doing. **(Marking)**

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.
EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Let's go back to substituting values. Who can look across the examples up here and explain what happened when you multiplied by -1? *(The sign of each number changed.)*
- Okay, the sign changed. Who can add on? *(The number that was smaller on one side of the number line is now bigger on the other side.)*
- How do we represent the new relationship with an inequality statement? *(We switch the > or < sign.)*
- Why do we switch the symbol? We need a reason to explain what's happening here. *(The numbers switched. So the order switches.)*
- Who can explain why it seems like we will have to change the inequality symbol? *(She said that the size of the numbers changed, so we have to change the inequality to show that the other number is bigger.)*

continued on next page

LESSON GUIDE 6

- Why does the inequality symbol change in this situation? *(You have to look at which number is to the right of the other. When we started, b was to the right of a, but when the numbers are opposites, -b is on the left of -a. It's smaller.)*
- That's an important point. We must consider which number is to the right of the other when we make our decision about which inequality symbol will make the statement true. **(Marking)**
- I notice that some of you added zero to the number line **(Marking)**, and that all of the values you chose were on the same side of zero. Both directions are changing, right? Then why does the inequality change? *(If they're positive, the number farther from zero is greater. Then when they "switch," it's now farther in the negative direction so it's less.)*
- Who can summarize what was just said about distance from zero?
- Who can use this same reasoning to explain what happens when both numbers are negative? *(You have to consider their distance from zero. The smaller number, which is farther from zero, becomes farther from zero in the positive direction and therefore is greater.)*
- So when both *a* and *b* are on the same side of zero, negating each of them actually changes which number is farther from zero, and that explains why the inequality symbol needs to be switched. **(Recapping)**
- Group B decided they needed to try something a little different. Let's hear about their choices for *a* and *b*. *(We chose one positive and one negative number for a and b. When we multiplied by negative numbers, the same thing happened, though. The numbers switched position on the number line. But they were not always farther from zero.)*
- Does this change our thinking about what happens when we multiply both sides of an inequality by a negative number? Say more about what happened on your number line when you chose the true inequality -2 < 5 as your original inequality, and then multiplied both numbers by -1. *(It's like the other groups saw. Both numbers moved to the opposite side of zero. So we switched and got 2 > -5.)*
- Who can explain what this group discovered about both sides? *(Even with one negative and one positive, negating both numbers switches the symbol.)*
- But why does the symbol switch? Is there a reason? *(Yes. The bigger is smaller again.)*
- I am confused. The bigger is smaller? Can someone be more specific? *(Okay. -2 is to the left of 5 on the number line. When we negate them, 2 is to the right of -5 on the number line.)*
- How does this relate to what we said earlier about the number "farther to the right" on the number line being the greater number? *(The number farther to the right is larger. We made our decision about the symbol using that idea.)*
- So, again we see that this idea of "farther to the right" is very important in our decision making about which inequality symbol makes the inequality true. **(Marking)**
- Can we be sure this will always happen, or is it only true for -2 and 5? Who else tried one negative and one positive number? What did you discover? *(It's always true. They always switch, just like when they were both on the same side of zero.)*
- Someone say why. *(If a is to the left of b, -a will be to the right of -b because negating both numbers moves each number to the other side of zero. So the numbers switch position on the number line. That means they switch size.)*
- That was a very important statement. Please repeat it. **(Marking)**
- Did you find that this was true for multiplication by all negative values, or just -1?
- Who can build on this to explain why the value doesn't have to be -1; that it can be any negative number? *(It's the negative that switches the numbers to the other side of zero, not the size of the number.)*

- It looks like we substituted different values for a and b and multiplied both sides of the inequality by various negative numbers. As a result, the sign of each number in the inequality changed, because negation moves the number to the opposite side of zero. This negation changed the left-right position of the numbers on the number line, and so the relative size of the numbers changed. What was the larger number in our original inequality is now the smaller number. **(Revoicing and Recapping)**
- Let's just clarify one more thing. We have been talking about the results of multiplying both sides of an inequality by a negative number. Do we need to be concerned about adding or subtracting negative numbers? Who can explain why adding or subtracting a -1 or -2 or -18 does not reverse the inequality? **(Challenging)** *(Both numbers move to the right or the left the same amount.)*
- Who agrees/disagrees?
- I'm hearing you mention direction **(Marking)**, but is the distance from zero also significant? *(The distance from zero changes, but by the same amount for each value. For example, both may move 4 to the left, so if one started a greater distance from zero, it will end up still a greater distance from zero.)*
- So adding or subtracting any number maintains the relative inequality, the inequality symbol. Multiplying by a positive number also maintains the relative inequality. We saw that in another task. Multiplying by a negative number reverses the inequality. We saw that in this task. **(Recapping)** What do we think happens when we divide by a negative number? Does that also "flip the inequality? **(Challenging)** Who can explain why division by a negative number also "flips" the inequality?

Application	Order the following numbers from least to greatest: -1, -50, 5, -49, -7.5, 48. Multiply each number by -1 and then re-order them from least to greatest. Explain any patterns that you notice.
Summary	See recapping above.
Quick Write	When you multiply both sides of $-7 < -6$ by -1, use the number line to explain why the inequality must be reversed.

TASK 7

Name____

Flip It Over–This Inequality is Done!

Two students, Dara and Jordan, are working as partners on an inequalities assignment. After working on a problem individually for a few minutes, they compare their work, shown below.

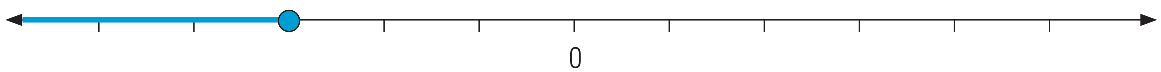
Dara:	Jordan:
$6 - 4x > 10$ $\underline{\quad + 4x > 10 + 4x}$ $6 > 10 + 4x$ $\underline{-10 - 10}$ $-4 > 4x$ $-\frac{4}{4} > \frac{4x}{4}$ $-1 > x$	$6 - 4x > 10$ $\underline{-6 \qquad -6}$ $-4x > 4$ $\frac{-4x}{-4} < \frac{4}{-4}$ $x < -1$

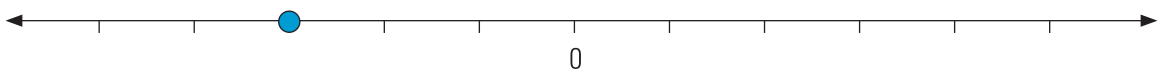
1. Are both algebraic approaches correct? Did both students end up with the same solution set? Why or why not?

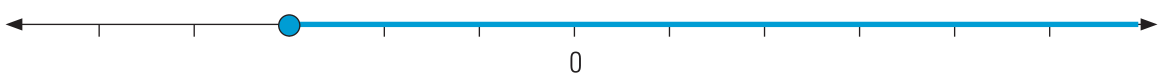
2. Solve the inequality $19 < -2 - 7x$. Explain your algebraic approach and then test one of the solutions by substituting it back into the inequality.

TASK 7

Which number line represents the solution set and why? Explain why the other number lines do not represent the solution set.







LESSON GUIDE 7

Flip It Over–This Inequality is Done!

Rationale for Lesson: In this task, students analyze two different algebraic approaches for solving inequalities with negative coefficients. They note that one approach maintains the negative coefficient, while the other does not. They analyze why it makes sense mathematically to reverse the inequality symbol only when the approach yields a negative coefficient.

Task 7: Flip It Over–This Inequality is Done!
Two students, Dara and Jordan, are working as partners on an inequalities assignment. After working on a problem individually for a few minutes, they compare their work, shown below.

1. Are both algebraic approaches correct? Did both students end up with the same solution set? Why or why not?
2. Solve the inequality $19 < -2 - 7x$. Explain your algebraic approach and then test one of the solutions by substituting it back into the inequality.

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP6 Attend to precision. MP7 Look for and make use of structure.	
Essential Understandings	• Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form $a > b$ (or $a \geq b$) where a and b are real numbers. • Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.	
Materials Needed	• Student reproducible task sheet • Calculators (optional)	

SET-UP PHASE

Read through the task before I have somebody read it aloud. Without giving away an answer, who can explain what you are asked to do in this problem? I'm going to give you about five to ten minutes of private think time to analyze the two solution strategies to determine if they are correct, as well as their similarities/differences. Be prepared to share your thinking with the group. Then you will use an algebraic method to solve the inequality that is provided in question 2.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Agrees with the way inequalities are solved, but is unsure what the solution(s) represent.	What does the solution $-1 > x$ mean in this problem?	Can you write one or more values that are solutions to the inequality? Consider the original inequalities–how do you know these values are solutions?
Claims that switching the inequality symbol is necessary and therefore only the second solution strategy is correct.	Can you tell me why the inequality symbol is reversed here?	Does dividing both sides of an inequality, say $4 < 8$, by 2 result in a correct inequality statement? What about -2? How is dividing by a negative value different?
Claims that the strategies result in different solution sets. $-1 > x$ is different than $x < -1$	Can you tell me about the solutions to each inequality?	What do you notice about each of your solution sets? Can you explain this?
Group finishes early.	Tell me what the difference is between Dara's and Jordan's solution paths.	Can you write a description of how you can solve any inequality?

LESSON GUIDE 7

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Determining the solution set to an inequality requires first determining the solution set to the associated equality because the equality divides the number line into two half lines (or half planes), only one of which makes the inequality a true statement. As a result, the properties of equality may be used in the process of solving an inequality.
EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Let's analyze Dara's and Jordan's solution paths. What did you notice about their solution paths? How are the solution paths the same/different? *(Dara and Jordan are doing the same problem. Dara adds, subtracts, and divides both sides. Jordan does the same thing, but she ends up dividing by a negative number and has to "switch" the inequality.)*
- Let's slow things down. Give me something that is the same first. *(They have the same inequality.)*
- They start out solving the same inequality. **(Marking)**
- Tell me some differences between the two students' solution paths. *(Dara moves the 4x to the other side of the inequality sign by doing the opposite of -4x; she adds +4x to each side.)*
- Someone say more. What did Jordan do differently? *(She moved the 6 to the other side.)*
- Someone who understands what Dara did tell us why she decided to move the 4x instead of the 6? Go up and point to the amounts. *(I didn't want to work with the negative 4x so if I took it to the other side, it would be positive.)*
- How many students understand why Dara moved the 4x? *(She didn't want to work with negatives like Jordan has here in her method.)*
- Tell us about the rest of Dara's work. For example, when she subtracts the 10 what happens? *(When she subtracts 10 from each side she is left with -4 on the left hand side because -10 + 6 is -4.)*
- So here, when we subtract 10 from each side, the -10 + 10 on the right side leaves us with 0 + 4x or just 4x , but leaves -10 + 6 or -4 on the left side. Why does adding 4x to each side eliminate the -4x from the left side and leave the whole 4x on the right side? **(Challenging)** *(On the left side, the -4x + 4x = 0 and 0 + 6 = 6. On the right side, the 4x and the 10 can't be combined because one has a variable but -10 + 6 have no variable.)*
- So they are like terms. **(Marking)**
- Explain the rest of Dara's work. *(She divides by 4 and this leaves -1> x.)*

<u>*The meaning of the solution set:*</u>

- What does the value x = -1 represent in this problem situation? *(The answer is the "breaking point" that we talked about. It divides the number line.)*
- What does this mean, "divides the number line"? *(The values on one side are solutions and the values on the other side are not.)*
- Who can say more about the meaning of "divides the number line"? *(If Dara and Jordan were solving equations, then x = -1 would be the solution to the equation.)*
- Is this true? Is x = -1 the solution to the equation? Turn and talk to your partners and see if you agree with the statement that x = -1 is the solution to the equation 6 - 4x = 10. *(Yes, we solved the equation and x = -1 is the solution.)*
- Is x = -1 the solution to $6 - 4x > 10$? *(No, the solution to the equation is what creates this "breaking point" that we talked about but we need to put in an inequality.)*
- So we do not just want to know the breaking point. *(No, we need to know which side of x = -1 works in 6 – 4x > 10. The values to the left work. The values less than negative 1.)*
- What do we mean when we say the side that "works?" What does it mean, to work? *(It means that the values on that side of x = -1 make 6 – 4x > 10 a true statement.)*

LESSON GUIDE 7

- Let's sum up what we have so far. It sounds like we can agree that the solution to the equation, in this case $x = -1$, divides the number line into two parts: one side will represent all of the solutions to the inequality. The other side will represent all of the values that are not solutions to the inequality. **(Recapping)**

Comparing -1 > x and x < -1 on the number line model:

- As I circulated, I noticed some confusion over the solutions $-1 > x$ and $x < -1$. Are these solutions saying the same thing? *(Yes.)*
- I heard a "yes," but who can convince us by showing the solutions on a number line? *(A student sketches a number line, shading values less than negative one.)*
- Who can explain how the number line drawn at the board represents the symbols? Point to the values on the number line as you explain. *(If negative one is greater than x, then x must be all of these values [pointing] to the left of negative one. In Jordan's solution, if x is less than negative one–it's all of these values to the left of negative one, which is the same thing.)*
- Can someone else put the ideas into their own words?
- We can see from the explanation provided and the number line drawn that the solutions $-1 > x$ and $x < -1$ represent the same solutions. We noted that Jordan and Dara used the properties of equality correctly and came to answers that appear different because of what they chose to add or subtract first and the division by a positive number versus a negative number, but the solution set is still the same. **(Revoicing and Marking)**

Comparing Jordan's and Dara's solution path:

- How did Jordan end up with a different way of writing the inequality? *(She never moved the -4x and because she didn't move it initially, then she had to divide by -4, which then "switches" the inequality symbol.)*
- Turn to a partner and say back what you heard.
- Someone recap for us. Say why one person is reversing the inequality symbol while the other is not. *(Because dividing by a negative gets opposites, the numbers switch on the number line. When we are on the other side of the number line, size works backwards. So > becomes < and < becomes >.)*
- Let's recap what we just talked about. As long as we're following the properties of equality correctly, we can solve an inequality in different ways because we maintain the balance and truth of the original statement. However, when multiplying or dividing by a negative number we need to change the direction of the inequality symbol in order to maintain this balance. **(Recapping)**

LESSON GUIDE 7

EU: Because a solution is a value for which the inequality is a true statement, substituting a solution for the variable into an inequality and simplifying will result in a true statement of the form *a* > *b* (or *a* ≥ *b*) where a and b are real numbers.

- We discussed this breaking point and how $x = -1$ divides the number line. How can we use the inequality to verify that our solutions are correct? *(We can go back and check the steps. If done right, we'll get the right answer.)*
- Who can add on to this? *(We can plug in values and see if the inequality works.)*
- Who agrees or disagrees? Who can add on by saying what it means for the inequality to "work"? *(It means it makes the symbols correct. You get a right answer…a true statement.)*
- I'm hearing that we can substitute a value back into the inequality to see if a true statement results. **(Revoicing)**
- Who can add on to what was just said by providing a value that we can substitute back into the inequality to make a true statement? *(We can use $x = -2$. Plug it in to Dara's inequality to get $14 > 10$. This is true so these values work.)*
- Who can demonstrate what we just discussed by trying another point?
- Do we need to try several points or is one point enough? *(We talked about how $x = -1$ divides the number line so we can test a point on each side of this "breaking point.")*
- Does everybody agree?
- Who can choose a value not in the solution set and demonstrate that the values on this side of the breaking point are not solutions? *(Let's choose $x = 5$. When you substitute $x = 5$ into the inequality, you get $-14 > 10$, which isn't true.)*
- Who agrees or disagrees?
- Does this mean ALL of the values not in the solution set will lead to untrue statements? *(Yes.)*
- Let's do a stop and jot as part of our summary. When we substitute a value that is not in the solution set we get an untrue statement. For example, name a value not in the solution set and write a description about why we would not get a true statement.
- Name some of your numbers that you used in your stop and jot. *(5, 6, 8, 10.)*
- So any value that is greater than -1 would make a false statement because the solution set says $x < -1$. **(Marking)**

Application	Solve the inequality $-5x - 7 \geq 23$. Represent your solution set in words and on the number line and then test a value for your solution set to verify that it is correct.
Summary	See recapping above.
Quick Write	No Quick Write for students.

Support for students who are English learners (EL):

1. In the Set-Up phase of the lesson, discuss what it means to compare the two solution strategies so students who are English learners may enter the task.
2. Consistently point to the parts of the inequality, as well as the different students' solution paths that are being referred to so that students can reference the algorithms and the spoken description. Use very specific references and avoid the use of "this" and "that" when describing the numbers.

Name____

TASK 8

Solution(s) to this Problem

1. How does finding the solution to the equation $3 - 2x = 11$ help you find the solution to $3 - 2x < 11$?

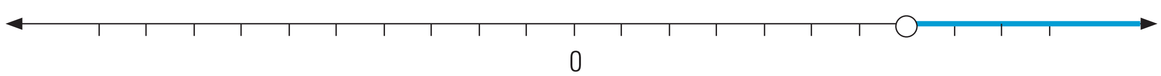
2. Which of the following have the same solution set? Justify your answer.

a. $3 - 2x < -11$

b. $-x + 4 < -3$

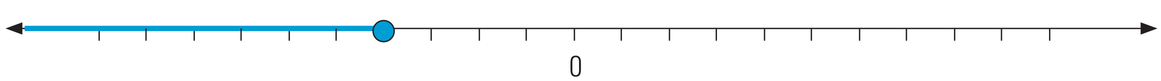
c. $-3 - 2x > 11$

d.



e. $77 < 11x$

3. Write BOTH an inequality and a context for which the number line below represents the solution:



LESSON GUIDE 8

Solution(s) to this Problem

Rationale for Lesson: Students solidify their understanding of solving inequalities with negative coefficients.

Task 8: Solution(s) to this Problem

How does finding the solution to the equation $3 - 2x = 11$ help you find the solution to $3 - 2x < 11$? Which of the following have the same solution set? Justify your answer.

a. $3 - 2x < -11$

b. $-x + 4 < -3$

c. $-3 - 2x > 11$

See student paper for complete task.

Common Core Content Standards	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Standards for Mathematical Practice	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and make use of repeated reasoning.
Essential Understandings	• Because an equation/inequality is a statement comparing two expressions, in order to preserve the comparison, certain rules must apply: a. adding or subtracting the same number to both expressions in the equation/inequality maintains the equation/inequality; b. multiplying or dividing both expressions in the equation/inequality by the same positive number maintains the equation/inequality; and c. multiplying or dividing both expressions in the inequality by the same negative number means that the inequality symbol must be reversed in order to maintain the inequality. • Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.
Materials Needed	• Student reproducible task sheet • Calculators (optional)

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SET-UP PHASE

Over the past several lessons, we've been analyzing inequalities. Now read over the task while I read it aloud. Work independently before comparing ideas with a partner.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Solves each equation/ inequality separately in Part 1.	Can you explain how you solved the equation and inequality? How was solving the equation the same/ different than solving the inequalities?	Is it possible to figure out the solutions to the inequality without necessarily solving it through the properties of equality? Explain.
Does not accurately identify letter c as the only one that does not match $x > 7$ in Part 2.	Can you explain your solution strategy to me? What steps did you take for solving each inequality?	The directions infer that there is one choice that is not correct. Which two inequalities look like they are most closely related? *(a & c)* Do you think the solutions to those two will be the same? Why or why not?
Group finishes early.	What was your approach to answering Part 2? How did you know which one did not belong?	Can you describe two different inequalities with the solution set shown in Part 3?

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SHARE, DISCUSS, AND ANALYZE PHASE

EU: Because an equation/inequality is a statement comparing two expressions, in order to preserve the comparison, certain rules must apply:

- **a. adding or subtracting the same number to both expressions in the equation/inequality maintains the equation/inequality;**
- **b. multiplying or dividing both expressions in the equation/inequality by the same positive number maintains the equation/inequality; and**
- **c. multiplying or dividing both expressions in the inequality by the same negative number means that the inequality symbol must be reversed in order to maintain the inequality.**

EU: Multiplying or dividing an inequality by a negative number reverses the position of the solutions to the inequality on the number line; therefore, the inequality symbol must be reversed in order to maintain the truth of the inequality.

- Let's zoom in on Part 2. Start with what you knew as soon as you looked at the choices a through e. *(We knew what the number line said. That said $x > 7$.)*
- How did you approach finding out if the inequalities had the solution $x > 7$? Can someone come explain their strategy starting with letter a: $3 - 2x < -11$? *(We started by subtracting 3 from both sides.)*
- Does everyone agree that you can subtract 3 from both sides? *(Yes.)* Why do we do this to both sides? *(Because it keeps the balance if you do it to both sides. The inequality doesn't change.)*
- What happens then, after you end up with $-2x < -14$? *(Then we divided by -2 on both sides.)*
- So you divided each side by negative 2. Did that leave you with $x < 7$? *(No!)* Why not? *(Because when you have a negative number that you are dividing by, the inequality reverses.)*
- Wait a minute, the inequality symbol reverses when we divide through by a negative number. **(Marking)** Why does the inequality symbol reverse? *(Because negatives mean the opposite of, so we have to turn it.)*
- Is everyone satisfied by that answer? I'm not sure we all are. Did anyone try substituting a value for x? How can that help us? *(Because if we try substituting a 2 for x in the original inequality, it is not true, but if we try 20, it is true. That means that the solution set is $x > 7$, not $x < 7$.)*
- Who can explain back what he just said? *(I heard him say that to prove that we needed to change the sign in order to keep the balance, we could just substitute two values to show if the < or > sign was needed. I also heard him say the solution set is NOT $x < 7$ but instead it is $x > 7$.)*
- Picking values and substituting to test which solution set is correct is a solid strategy for determining if our solution set is correct.
- So in this case we did need to reverse the inequality symbol when we divided through by a negative number. **(Marking)** Do we have to reverse the symbol when we use any operation with negative numbers? *(No, only with multiplication and division.)*

<u>Planning ahead to avoid reversing the inequality:</u>

- I wonder if there is a way to solve this inequality without having to reverse the inequality symbol. Is that possible? Take a minute and turn and talk with a partner. Figure out a way to solve $3 - 2x < -11$ without reversing the symbol.
- Okay, I see that some groups are ready to share. Come up and walk us through how you solved this inequality without needing to divide through by a negative 2. *(We actually added -2x to both sides first. That gave us $3 > -11 + 2x$. Then we added 11 to both sides. That gave us $14 > 2x$. When we divided each side by 2 we got $7 > x$.)*

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- Can someone say back why this group did not have to reverse the inequality symbol to solve the inequality? *(We only reverse the inequality symbol when multiplying or dividing through by a negative number. Because they move the x instead of the plain number, they got rid of the negative that was what we divided by.)*
- So we can divide by a negative number and reverse the inequality symbol. Or we can eliminate the need to do that by applying the properties of equality in order to just multiply or divide by positive numbers. **(Revoicing and Recapping)**
- Can we really add x's to each side? *(Yes, it does not matter what we add or subtract as long as we do it to both sides because that is how we keep the inequality true.)*
- Will this strategy of dealing with the x's instead of the numbers first to eliminate the need to reverse the inequality symbol later work for letter b: $-x + 4 < -3$? Work at your groups to see if you can use this same type of reasoning when you apply this strategy.
- Come explain how this strategy worked for letter b. *(We added x to both sides. That gave us $4 > -3 + x$. Then we added 3 to each side. That gave us $7 > x$. It worked.)*
- Let's compare that to how most of you originally solved it. Explain how you solved $-x + 4 < -3$ when you needed to reverse the symbol. *(We subtracted 4 from each side. That gave us $-x < -7$. Then we divided through by -1 and reversed the symbol so that we got $x > 7$. The answers were the same!)*
- Let's look a letters a and c. They looked incredibly similar. There were similar numbers and operations. Do you think that they have the same solution set? Why or why not?
- We know from our discussion that letter a *($3 – 2x < -11$)* does match the number line in letter d. Does letter c *($-3 – 2x > 11$)*? *(Yes, they'll be the same because it's negative 11 so the sign will reverse and they'll be the same.)*
- The negative 11 reverses the inequality symbol? Can someone else respond to this? *(That's not true–you only reverse the inequality symbol when you divide or multiply by a negative. This is not attached to a variable so that does not really work.)*
- Can someone else say back why the symbol does not reverse based on the -3? *(Well, we have to be multiplying or dividing by a negative but that is not the case with the -3. BUT we will need to reverse the symbol when we divide through by negative 2.)*
- Wait, so we may need to reverse the inequality symbol? How can that be? We just said that we did not have to. *(The -3 does not cause the sign to reverse but if we add 3 to each side we end up with $-2x > 14$. Then we have to divide through by -2. That's when the symbol changes.)*
- Someone come up to the board/overhead and write out the way that was just explained. Be sure to explain why each step is working.
- So we end up with $x < -7$. Does this match the solution set in a and number line in c? *(No. That's the only one that doesn't match.)*
- Can we solve letter c without having to reverse the symbol? *(Sure, if we added 2x to each side first.)* Let's all take a minute to solve it that way.
- So today we used two strategies; in one we eliminate the reversing of the symbol by adding or subtracting the variable quantity so we are left with a positive variable, and in the other we left the negative x's on the original side but reversed the symbol when we multiplied/divided by a negative. **(Recapping)** Which strategy worked better for you? *(It doesn't matter which one since both strategies yielded the same solution set. But when you use a strategy that gets to multiplying or dividing through by a negative, you have to remember that it reverses the inequality symbol.)*
- Are there always multiple ways that we can approach solving an inequality? **(Challenging)** Let's use some of your inequalities from part c to support your answer to this question.

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Application	No application.
Summary	What have we learned about solving inequalities? What do we know about inequality solutions?
Quick Write	Describe several similarities and differences between equations and inequalities.

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